

ASTRONOMY 8400 – SPRING 2026  
Homework Set 4 – Answers

1.a)

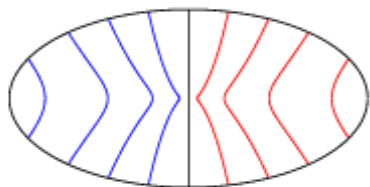
$$M(R) = \sigma\pi R^2 \quad \text{where } \sigma = \text{mass/area} = \text{constant}$$

$$v(R) = \sqrt{\frac{GM}{R}} = \sqrt{\frac{G\sigma\pi R^2}{R}} = \sqrt{G\sigma\pi R}$$

$$b) \quad v_r = v(R) \cos\phi \sin i = \sqrt{G\sigma\pi} \sqrt{R} \cos\phi \sin i$$

$$c) \quad R = k \frac{v_r^2}{\cos^2\phi} \quad \text{where } k = \text{const.}$$

Isovelocity plot using IDL and  $R(\Phi)$  for different  $v_r$  is given below. Note that the contours are properly spaced for equal intervals of  $v_r$  ( $R \propto v_r^2$  at  $\Phi=0$ ). (However, I didn't count off if they weren't.)



2. This involves the “radius of influence” equation for a SMBH

a) The resolution of *HST* in the optical is  $\sim 0.1''$ , which corresponds to a projected linear distance of  $1.50 \times 10^{19}$  cm ( $= 4.85$  pc) at a distance of 10 Mpc. So:

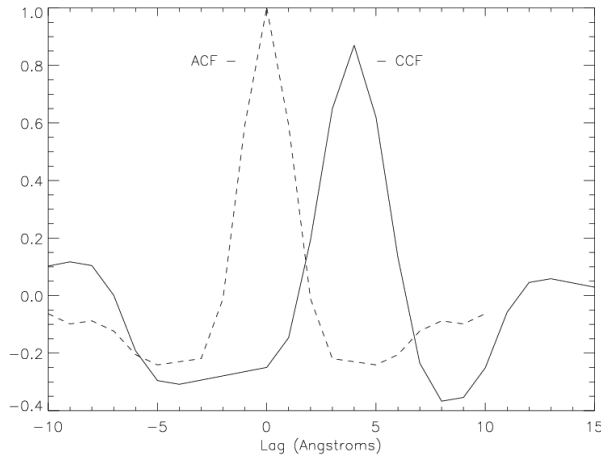
$$M_{\bullet} > \frac{r\sigma_*^2}{G} = 9.0 \times 10^{40} \text{ g} = 4.5 \times 10^7 M_{\odot}$$

b) At  $1''$ , the projected linear size at 10 Mpc is  $1.50 \times 10^{20}$  cm ( $= 48.5$  pc), and the minimum mass is  $4.5 \times 10^8 M_{\odot}$ .

c) From the graph of Kormendy et al., the minimum bulge luminosity in absolute B magnitude is  $M_B \approx -18$  ( $\sim 10^9 L_{\odot}$ ) for the *HST* case and  $M_B \approx -20.5$  ( $\sim 10^{10} L_{\odot}$ ) for the optical case (with large dispersions for both). These correspond to moderate-size ellipticals and bulges (like the Milky Way) for the former and large ellipticals and bulges for the latter.

d) From the Gebhardt et al. correlation,  $M = 1.5 \times 10^8 M_{\odot}$ . For the case above, you would probably detect the SMBH with *HST*, but not from the ground.

3. a) b) I calculated the CCF and ACF using the IDL procedure C\_CORRELATE. The CCF is somewhat sensitive to the “window” used, which is the length of spectrum on either side. If you add a bunch of 1.0s on either side of the spectra (I added some) it should be symmetric. Here’s the CCF for the star/galaxy, and the ACF for the star.



c) CCF peak is at 4Å:

$$v_r = \frac{\Delta\lambda}{\lambda} c = \frac{4}{8542} (3.0 \times 10^5) = 140 \text{ km s}^{-1} \text{ for the galaxy.}$$

You get the same answer from the centroids (in this case minima) of the absorption lines.

d) This wasn’t a great question, since the profiles are not really Gaussians. The FWHM is 3Å (105 km s<sup>-1</sup>) for the galaxy and 2Å (70 km s<sup>-1</sup>) for the star. Assuming they are Gaussians, and the stellar profile gives the line-spread function (LSF), the intrinsic FWHM of the galaxy absorption line is:

$$\text{FWHM}_{\text{intr}} = \sqrt{\text{FWHM}_{\text{obs}}^2 - \text{FWHM}_{\text{LSF}}^2}$$

$$\text{FWHM} = \sqrt{(105)^2 - (70)^2} = 78 \text{ km s}^{-1}$$

The intrinsic velocity dispersion is

$$\sigma = \text{FWHM} / 2.355 = 33 \text{ km s}^{-1}$$

Note that the cross-correlation of two spectra with many features is usually done with the wavelength scale in  $\ln\lambda$ . The reason is that for a given radial velocity,  $\lambda \propto \Delta\lambda$ , so the shift in wavelength is not constant. However  $\Delta\ln\lambda$  is:

$$\Delta\ln\lambda = \frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$$