ASTRONOMY 8400 – SPRING 2024
Homework Set 2 – Answers

1.a)

$$\frac{\dot{M}}{M} = \frac{L}{\eta c^2}\frac{4\pi G c m_p}{\sigma_e L_E} = \frac{4\pi G m_p}{\sigma_e c}\frac{L}{\eta L_E}$$

$$\frac{\dot{M}}{M} = 7.02 \text{ x } 10^{-17}\ \frac{L}{\eta L_E}\ \ \text{sec}^{-1}$$

1.b)

If $\eta = 0.1$ and $L / L_E = 1$:

$$\frac{\dot{M}}{M} = \frac{dM/dt}{M} = 7.02 \text{ x } 10^{-16}\ \text{sec}^{-1}$$

$$\int_{M_i}^{M_f} \frac{dM}{M} = 7.02 \text{ x } 10^{-16} \int_0^{\tau} dt$$

$$\tau = 1.42 \text{ x } 10^{15} \ln(M)\Big|_{10^6}^{10^8} = 1.42 \text{ x } 10^{15} \ln\left(\frac{10^8}{10^6}\right)$$

$$\tau = 6.56 \text{ x } 10^{15}\ \text{sec} = 2.08 \text{ x } 10^8\ \text{yrs}$$

2. a)

$$z = \lambda_0/\lambda_1 - 1 = 5500/1500 - 1 = 2.548$$

$$v = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} c = 0.8528c = 255{,}670 \text{ km s}^{-1}$$

b) From Ned Wright's cosmology calculator
(http://www.astro.ucla.edu/%7Ewright/CosmoCalc.html):
Lookback time for current cosmology = 11.1 Gyr
Age of the Universe = 13.7 Gyr
We see the quasar at age: 13.7 – 11.1 = 2.6 Gyr

2.c)

In the rest frame of the quasar, the line is centered at 1550 A

The far blue wing is at:

$\lambda_1 = \lambda_0/(1+z) = 5200\ /\ (3.548) = 1466\ \text{A}$

Using the nonrelativistic Doppler shift in the rest frame of the quasar:

$$v = \frac{\Delta\lambda}{\lambda_{lab}} c = \frac{1550 - 1466}{1550} = 0.0542c = 16{,}250\ \text{km s}^{-1}$$

Using the relativistic Doppler shift in the rest frame of the quasar:

$$v = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} c = 0.0527c = 15{,}800\ \text{km s}^{-1}$$

(negative if viewed outside of the AGN)

3. $$\nu = \frac{c}{\lambda} \rightarrow \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$F_\lambda d\lambda = F_\nu d\nu \rightarrow F_\lambda = F_\nu \frac{d\nu}{d\lambda} = \frac{c}{\lambda^2} F_\nu$$

$$F_\nu \propto \nu^{-\alpha_\nu}$$

$$F_\lambda \propto \frac{1}{\lambda^2} \nu^{-\alpha_\nu} \propto \frac{1}{\lambda^2}\left(\frac{1}{\lambda}\right)^{-\alpha_\nu} \propto \lambda^{-2}\lambda^{\alpha_\nu}$$

$$F_\lambda \propto \lambda^{\alpha_\nu - 2} \propto \lambda^{-\alpha_\lambda}$$

$$\text{Thus: } \alpha_\lambda = 2 - \alpha_\nu$$

$$F_\nu\left(\frac{\text{ergs}}{\text{sec cm}^2\ \text{Hz}}\right) = F_{ph}\left(\frac{\text{photons}}{\text{sec cm}^2\ \text{keV}}\right)(h\nu)$$

$$F_{ph} \propto E^{-\Gamma} \propto \nu^{-\Gamma} \propto \nu^{-1} F_\nu \propto \nu^{-1}\nu^{-\alpha_\nu}$$

$$\nu^{-\Gamma} \propto \nu^{-\alpha_\nu - 1}$$

$$\text{Thus: } \Gamma = \alpha_\nu + 1$$

4. For $L_\nu = \nu^{-\alpha_\nu}$:

$\alpha_\nu(10\text{ cm - }100\ \mu\text{m}) = -1.1$ (sharp drop to mm range)

$\alpha_\nu(100\ \mu\text{m- }1\ \mu\text{m}) = 1.0$ (relatively flat)

$\alpha_\nu(1\ \mu\text{m - }2500\text{ A}) = 0.75$

$\alpha_\nu(2500\text{ A - }2\text{ keV}) = 1.5$

$\alpha_\nu(2\text{ kev - }100\text{ keV}) = 0.7$

Distance to Seyfert: $d = \dfrac{cz}{H_0} = 120\text{ Mpc}$

$L_\nu(5100) = 4\pi d^2 F_\nu(5100) = 7.47\times10^{28}\text{ ergs s}^{-1}\text{ Hz}^{-1}$

Connecting the power laws and scaling to match this point gives the plot below.

Integrating $L_\nu(\nu)$ gives: $L = 3.7\times10^{44}\text{ ergs s}^{-1}$

Integrating that part above 13.6 eV: $L = 7.6\times10^{43}\text{ ergs s}^{-1}$

(Ionizing flux is ~20% of the total flux.)