

Gas Dynamics

- Gaseous nebulae are not static entities:
- Expansion of ionized gas into the ISM (Planetary Nebula)
- Radiation driving of ionized clouds (stellar winds and AGN)
- Gravitational motions around a supermassive black hole (AGN)
- Expansion of ionized “sphere” of gas after a hot star turns on (H II region)
 - creates an ionization/shock front
- Explosions into the surrounding ISM (novae and supernovae)
 - create shock fronts

Shock Fronts

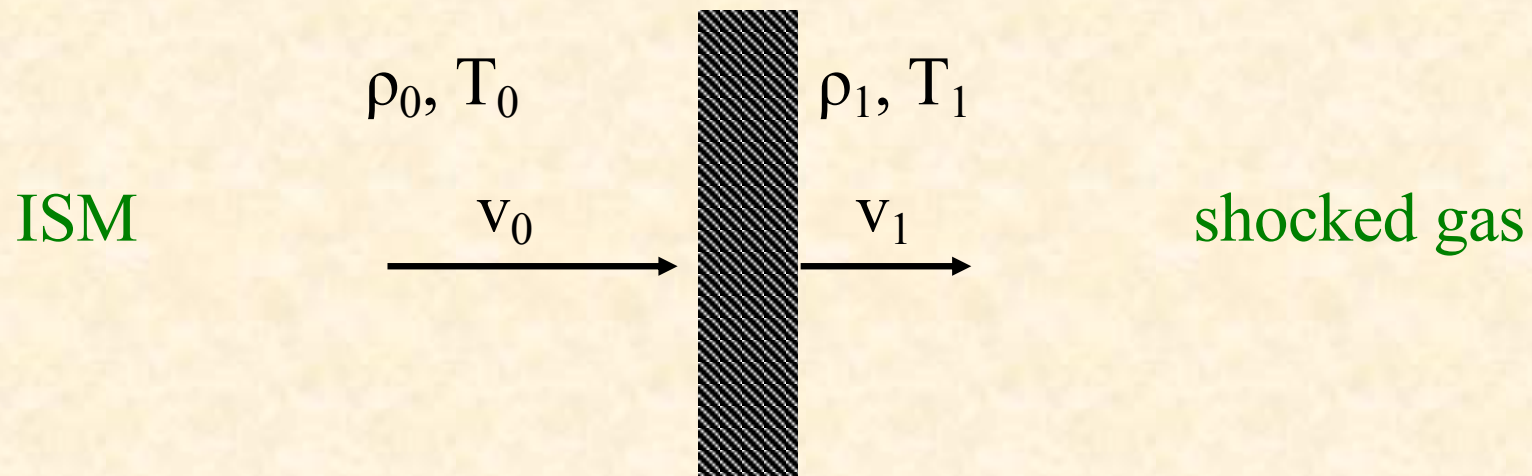
Ex) Piston in a tube of gas (adiabatic case)



- Moving piston starts a compression wave (higher P, ρ).
- Speed of sound is higher in compressed region.
- Gas in the compressed region travels much faster, acting to steepen the pulse.
- A nearly discontinuous shock front is formed
- Note that velocity of piston must be supersonic (compared to unshocked gas), or the disturbance will spread out (no more shock front).

Shock Fronts

- Ex) Explosion of a Supernova into the ISM
- Supernova remnant moves at supersonic speed in the ISM at v_0
- Builds up a pulse of increased pressure
- Pulse steepens because sound velocity is higher in the compressed region. In this case, gas can escape out the back end.
- Consider a shock front propagating through the ISM:
 - Reference frame: traveling with the shock



ρ – mass density, T – temperature, v - velocity

Conservation laws

1) $\rho_0 v_0 = \rho_1 v_1$ (cons. of mass)

2) $P_0 + \rho_0 v_0^2 = P_1 + \rho_1 v_1^2$ (cons. of momentum)

I. Adiabatic Case (e.g., initial SNR explosion)

For adiabatic expansion (no radiation loss from compression):

$$P = K\rho^\gamma \quad (\gamma = 5/3 \text{ for monatomic gas})$$

"It can be shown that (see Osterbrock, p. 162)":

$$3) \quad \frac{1}{2} v_0^2 + \frac{\gamma}{\gamma - 1} \frac{P_0}{\rho_0} = \frac{1}{2} v_1^2 + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} \quad (\text{cons. of energy})$$

$$\frac{1}{2} v_0^2 + \frac{5}{2} \frac{P_0}{\rho_0} = \frac{1}{2} v_1^2 + \frac{5}{2} \frac{P_1}{\rho_1} \quad (\text{for monatomic gas})$$

(1st term - flow kinetic energy per mass, 2nd term - thermal kinetic energy plus compression energy)

II. Isothermal Case (e.g., stellar wind bubble inside H II region)

- gas just ahead and just behind shock has same temperature ($T_0 = T_1$)
- applies to shocks *within* H I or H II regions since the heating and cooling time scales are much smaller than the expansion time scales (works also for planetary nebula)

$$P = \frac{\rho k T}{\mu m_H} \quad (\mu - \text{atomic weight, } T = \text{const.})$$

$$P \propto \rho \quad (\gamma = 1)$$

So for the conservation of energy :

$$3) \quad \frac{P_0}{\rho_0} = \frac{P_1}{\rho_1} = \frac{kT}{\mu m_H}$$

The sound speed in the undisturbed gas is given by :

$$c_0 = \sqrt{\frac{\gamma P_0}{\rho_0}} = \sqrt{\frac{\gamma k T_0}{\mu_0 m_H}} \quad (\sim 1 \text{ km s}^{-1} \text{ for } T = 100 \text{ K})$$

and the Mach number of the shock front is :

$$M = \frac{|v_0|}{c_0} \quad \text{Supersonic if } M > 1$$

Combining the conservation equations, we get :

$$1) \frac{P_1}{P_0} = \frac{2\gamma}{\gamma+1} M^2 - \frac{\gamma-1}{\gamma+1}$$

$$2) \frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2 + 2}$$

Ex) For a strong shock (large M), $P_1/P_0 \rightarrow \infty$

Adiabatic monatomic case: $\rho_1/\rho_0 \rightarrow 4$, $v_1 \rightarrow \frac{1}{4} v_0$

Isothermal case: $\rho_1/\rho_0 \rightarrow \infty$, $v_1 \rightarrow 0$

Ionization Fronts

- Ex) An O or B star turns on
- An H II region expands at the rate that ISM is ionized
- The momentum conservation law is the same. The degree of ionization changes sharply across the boundary:

$$1) \rho_0 v_0 = \rho_1 v_1 = m_i \Phi_i \quad (\text{cons. of mass})$$

where m_i = mass of ionized gas per created ion / electron pair

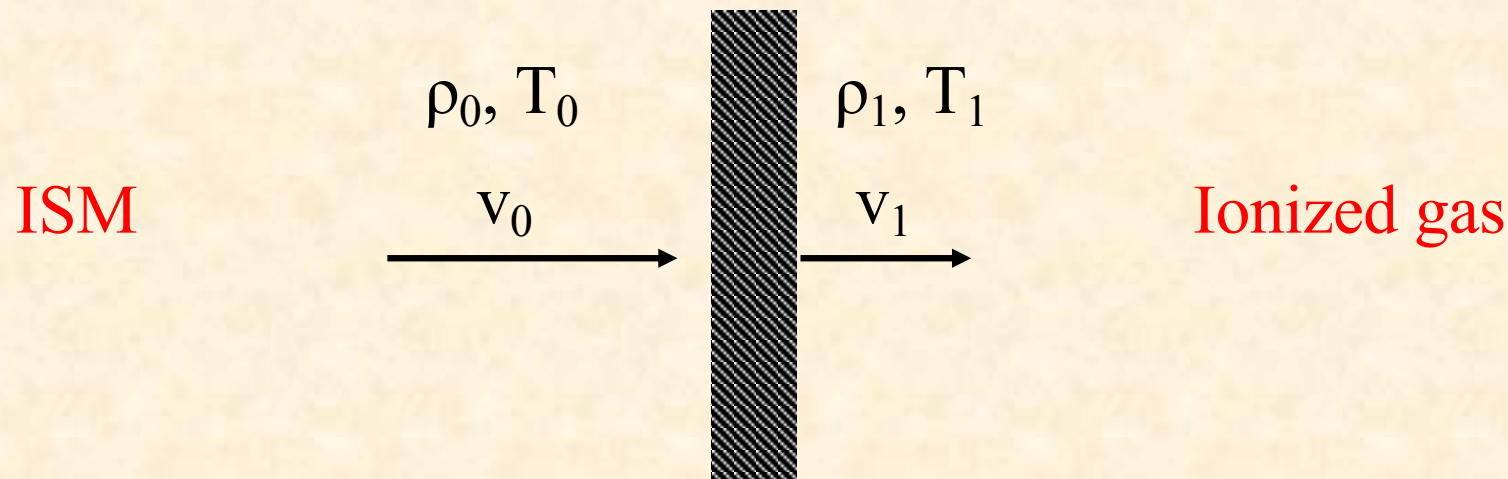
($m_i = m_H$ for a pure H nebula)

$$\Phi_i = \int_{\nu_0}^{\infty} \frac{F_\nu}{h\nu} d\nu = \text{flux of ionizing photons}$$

Ionization Fronts (H II Region)

For an ionized nebula, we have two very different temperatures. The conservation of energy becomes:

$$3) \frac{P_0}{\rho_0} = \frac{kT_0}{\mu_0 m_H} \quad \text{and} \quad \frac{P_1}{\rho_1} = \frac{kT_1}{\mu_1 m_H} \quad \begin{array}{l} (T_1 \text{ fixed by photoionization,} \\ T_0 \text{ is temperature of ISM)} \end{array}$$



Solving for the density ratio :

$$\frac{\rho_1}{\rho_0} = \frac{c_0^2 + v_0^2 \pm \left[(c_0^2 + v_0^2)^2 - 4c_1^2 v_0^2 \right]^{1/2}}{2c_1^2} \quad (\text{Osterbrock, p. 166})$$

There are two allowed ranges for v_0 (speed of ionization front),

since $\frac{\rho_1}{\rho_0}$ must be real :

$$1) v_0 \geq c_1 + \sqrt{c_1^2 - c_0^2} \equiv v_R \approx 2c_1 \quad (\text{for } c_1 \gg c_0 \text{ in H II region})$$

v_R is the velocity of an "R - critical front" (R - rare or low density),
since as $\rho_0 \rightarrow 0$, $v_0 \rightarrow \infty$ and therefore exceeds v_R

For $v_0 > v_R$, an R - type front moves supersonically into the ISM

$$2) v_0 \leq c_1 - \sqrt{c_1^2 - c_0^2} \equiv v_D \approx \frac{c_0^2}{2c_1} \quad (c_1 \gg c_0)$$

v_D is the velocity of a "D - critical front" (D - dense),

When $v_0 < v_D$, a D-type front moves subsonically into the ISM

Consider an H II region expanding into the ISM

In this case, c_0 is small, and $v_0 \gg c_1$, so again there are two cases :

$$1) \frac{\rho_1}{\rho_0} = \frac{v_0^2}{c_1^2} \left(1 - \frac{c_1^2}{v_0^2} \right) \gg 1 \quad (\text{strong R - type front})$$

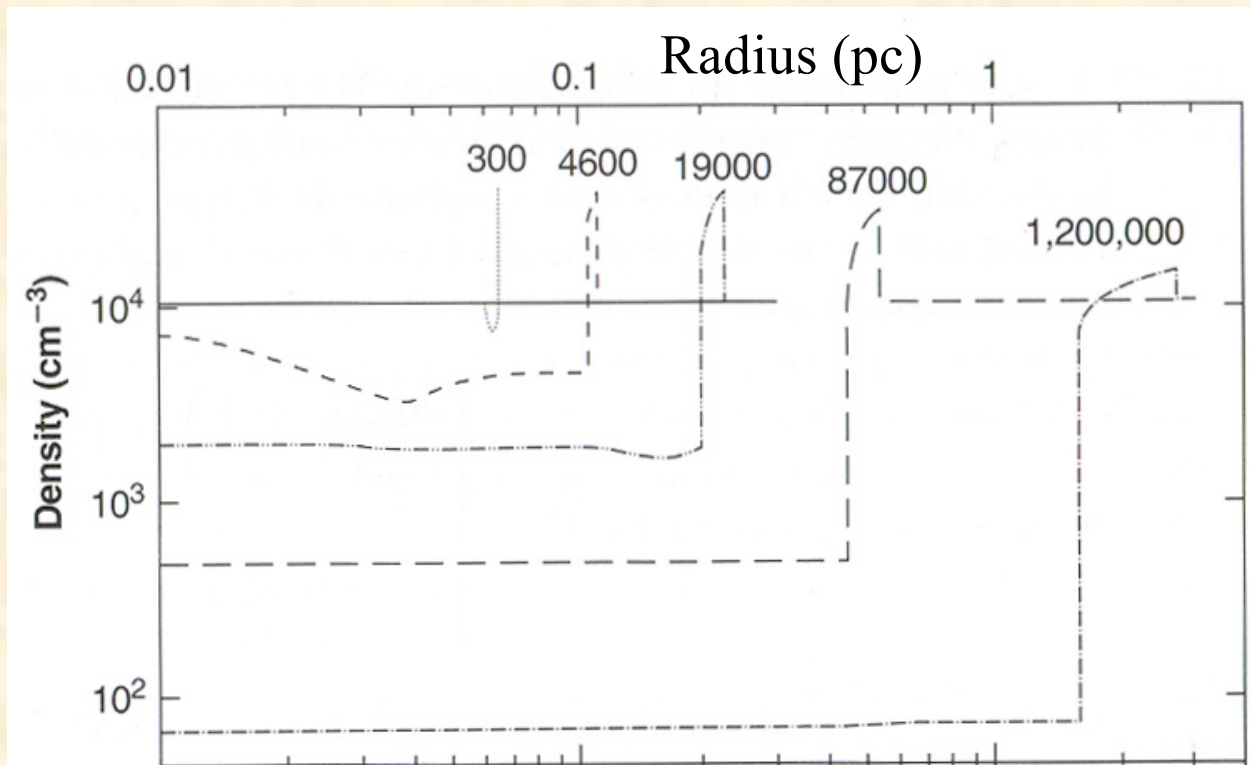
$$2) \frac{\rho_1}{\rho_0} = 1 + \frac{c_1^2}{v_0^2} \approx 1 \quad (\text{weak R - type front})$$

Case 1) is not physical, since high densities in the ionized gas disrupt the shock. Using the conservation of mass for case 2):

$$v_1 = v_0 \left(1 - \frac{c_1^2}{v_0^2} \right) \approx v_0 \gg c_1$$

So the ionization front initially moves supersonically into the ISM, due to the large number of ionizing photons.

Model of Expanding H II Region around an O Star



(Osterbrock & Ferland, p. 167)

- $t = 0 - 300$ years: weak R-type ionization front moves out at $v_0 \approx 300$ km/sec
- $t = 300 - 4600$: Φ_i decreases as the front expands, due to geometric dilution and photoionization; v_0 decreases until it reaches $v_R \approx 2c_1$
- $t \approx 4600$ years: a shock front breaks off and compresses the gas ahead of the ionization front, which becomes D-type; $v_0 \approx v_D$, $v_1 \approx c_1$
- $t = 4600 - 1$ million years: the ionization front continues as D-type, which slows down until it terminates at the Stromgren radius.